

The Antimatter Photon Drive

A Relativistic Propulsion System

Darrel Smith* & Jonathan Webb†
Embry-Riddle Aeronautical University
Prescott, AZ 86301

This paper describes a propulsion system that derives its thrust from electron-positron annihilation. It also describes how a spacecraft equipped with this engine can be used to launch manned missions to Mars and Jupiter in as little as 3.8 and 10.8 days respectively. Technical problems associated with the antimatter engine and potential solutions are also discussed. Throughout this paper, this engine is referred to as the Antimatter Photon Drive or (APD).

Nomenclature

a	= acceleration
c	= the speed of light in a vacuum (3.00×10^8 m/s)
e^+	= positron
e^-	= electron
g	= gravitational acceleration (9.81 m/s^2)
I_{sp}	= specific impulse
m_p	= mass of propellant
\dot{m}	= mass flow rate of propellant
M_i	= initial mass of spacecraft
M_f	= final mass of spacecraft
p	= proton
\bar{p}	= antiproton
t_t	= total time of flight relative to the earth
t_o	= total time of flight relative to a passenger in the spacecraft
ΔV	= change in velocity
β	= v/c
γ	= gamma ray, also $\gamma = \frac{1}{\sqrt{1-\beta^2}}$
μ^+	= positively charged muon
μ^-	= negatively charged muon
π^+	= positively charged pion
π^-	= negatively charged pion
π^o	= neutrally charged pion
ν_μ	= muon neutrino
ν_e	= electron neutrino
$\bar{\nu}_\mu$	= muon antineutrino
$\bar{\nu}_e$	= electron antineutrino
$m_e c^2$	= mass-energy of an electron (0.511 MeV)

Introduction

The efficient energy production resulting from matter-antimatter annihilations has been discussed since antiprotons were first produced in 1955. Methods for converting $\bar{p}p$ annihilation energy into thrust to

develop advanced rocket motors continues to *spark* the imaginations of physicists and engineers.¹⁻³ Motor designs using antiprotons include inertial confinements fusion (ICF) (i.e., using the resulting muons to catalyze D T fusion), as well as venting the resultant charged particle through magnetic nozzles to create high specific impulses with low thrust.

One major problem arises when trying to construct a rocket engine based on $\bar{p}p$ annihilation. Much of the energy is converted into mass which reduces the momentum of the outgoing particles, thus reducing the thrust. The most typical reaction that occurs in $\bar{p}p$ annihilation (near rest) is



where n number of pions (π^+ , π^- , and π^o 's) are produced in the final state. There are typically 3-7 pions produced in $\bar{p}p$ annihilations near rest. The amount of kinetic energy released in the annihilation is 1876 MeV (i.e., $2m_{proton}c^2$) minus the rest mass energy of the pions (~ 140 MeV for each pion). The pions have a long enough decay length that most of their kinetic energy can be used for momentum transfer (i.e., thrust). Table 1 shows the mean decay lengths ($\beta\gamma c\tau$) for pions and muons resulting from $\bar{p}p$ annihilations and subsequent pion decays. The mean pion momentum is ~ 350 MeV/c for a mean number of 5 pions in the final state. The momentum is converted into the relativistic factor $\beta\gamma$ and this is multiplied by the factor $c\tau$, the mean lifetime of the particle multiplied by the speed of light. The product of $\beta\gamma$ times $c\tau$ gives the mean decay lengths shown in Table 1.

In the design of any $\bar{p}p$ engine, the pion momentum must be captured before a significant fraction of them decay. If the pion momentum (π^+ , π^-) is not extracted within 14 meters, then $\sim 63\%$ of them will decay into muons, whose smaller momentum can then be extracted. The momentum released in neutrinos (ν 's) cannot be captured because of their extremely low cross-section. Finally, the π^o 's rapidly decay into

*Ph.D Research Advisor, College of Arts and Sciences

†Undergraduate Student, College of Engineering

Copyright © 2001 by the American Institute of Aeronautics and Astronautics, Inc. All rights reserved.

Decay Process	$c\tau$	$\beta\gamma c\tau$
$\pi^\pm \rightarrow \mu^\pm \nu_\mu$	7.80m	19.4m
$\pi^o \rightarrow \gamma\gamma$	25.1nm	0.62nm
$\mu^\pm \rightarrow e^\pm \nu_\mu \nu_e$	659m	188m

Table 1 This table shows the mean decay length of pions and muons assuming a mean pion and muon momentum of 350 MeV/c and 30 MeV/c respectively.

two gamma rays (\sim 67 MeV each) and have a high efficiency for imparting momentum.

The radioactive decay of pions (π^+, π^-) shown in Table 1 puts constraints on the size of the absorber. A significant fraction of the $\bar{p}p$ center-of-mass energy is converted into mass ($m_\pi = 140$ MeV/c), thus reducing the momentum transfer to the absorber. However, there is another form of antimatter annihilation that produces \sim 100% gamma rays resulting in a much improved momentum transfer, and likewise more thrust. This is the electron-positron annihilation process

$$e^+e^- \rightarrow \gamma\gamma . \quad (2)$$

The electron (e^-) positron (e^+) annihilation produces two gamma rays (0.511 MeV each) 99% of the time. Since none of the center-of-mass energy is converted into mass, the full center-of-mass energy is available for thrust where $p = E/c$ for γ rays. The gamma rays are electrically neutral and emitted in random directions. Because they are electrically neutral, it is impossible to focus them with electromagnetic fields, whereas the charged π^+ and π^- particles from $\bar{p}p$ annihilation can be focussed to improve the momentum transfer.

Various Rocket Fuels

The energy and propulsive advantages of antimatter are shown in Table 2.⁴ So far, the most efficient and yet unattained chemical rocket propellants release 4.77×10^8 J/kg of energy. Furthermore, the table shows that ^{235}U fission releases nearly 1.72×10^5 times more energy than chemical reactions, D³He fusion produces over 7.80×10^5 times more energy, while matter-antimatter fusion produces over 1.88×10^8 times more energy than chemical rocket propellants (e.g., metastable helium).

Matter-antimatter annihilations yield the highest specific impulse and jet power of any propulsion system. Furthermore, it can yield specific impulses on the order of 10^7 seconds with thrust in the mega-Newton range (see Fig. 1). It stands out as the most efficient of all known propulsion systems.

Theory of electron-positron propulsion

A spacecraft equipped with an APD engine will annihilate electrons and positrons to produce gamma rays that will be captured in a parabolic dish (Fig. 2).

Yields from Various Energy Sources

Fuels	Energy Release (J/kg)	Converted Mass Fraction
<u>Chemical</u>		
LO/LH	1.35×10^7	1.25×10^{-10}
Atomic Hydrogen	2.18×10^8	2.40×10^{-9}
Metastable Helium	4.77×10^8	5.30×10^{-9}
<u>Nuclear Fission</u>		
^{235}U	8.20×10^{13}	9.10×10^{-4}
<u>Nuclear Fusion</u>		
DT (0.4/0.6)	3.38×10^{14}	3.75×10^{-3}
CAT-D T(1.0)	3.45×10^{14}	3.84×10^{-3}
D ³ He (0.4/0.6)	3.52×10^{14}	3.90×10^{-3}
p B ¹¹ (0.1/0.9)	7.32×10^{13}	8.10×10^{-4}
<u>Matter-Antimatter</u>		
	9×10^{16}	1.00

Table 2 Energy released by various fuels. Weight compositions correspond to mixture ratios for fusion fuels, for example, (0.4/0.6) for the D T fuels.⁴

The e^+e^- beams are directed towards a collision point at the focus of the parabolic dish placed at the extreme end of the spacecraft. Each annihilation will release two γ -rays back-to-back where one or both photons will impact the parabolic dish depending upon the extent of the dish. Two possibilities are considered when the photon impacts the parabolic dish. In the first case, the photons are completely absorbed; while in the second case, they are totally reflected. Since the γ rays are neutral and cannot be focussed, the choice of a totally reflecting parabolic dish becomes the obvious choice. While a totally reflecting dish for γ rays at these energies is beyond current technology, this particular design is considered because it describes the maximum thrust available from e^+e^- collisions.

Theory

Velocity Boosts Using Lorentz Transformations

In this section, the relativistic velocity of the space craft is calculated as a function of its fuel. As the spacecraft receives successive boosts from one of the two γ rays, it is boosted continuously from inertial frame to inertial frame.

For the purpose of this derivation, it is assumed that the spacecraft is moving with an initial velocity V with respect to the earth's inertial frame. A small amount of mass dm is annihilated to produce the γ rays used to propel the rocket. As a result of the boost, the spacecraft's mass is reduced by dm while its velocity increases to $V + dv$. The Lorentz transformation relating velocities between the earth's frame and the

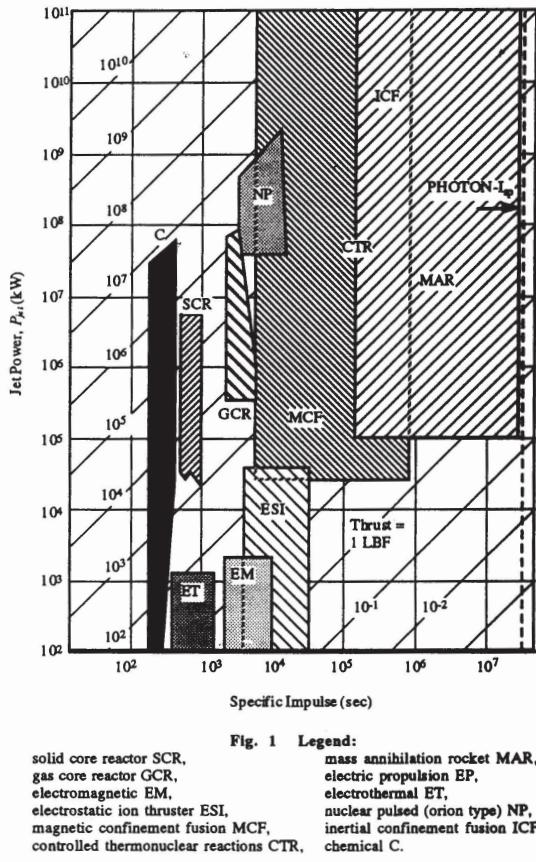


Fig. 1 This figure shows the specific thrusts resulting from various propulsion engines.⁴

spacecraft's frame is

$$v = \frac{v' + V}{1 + \frac{v'V}{c^2}} \quad (3)$$

where v' is the velocity of the spacecraft measured in the inertial frame traveling at velocity V with respect to earth's inertial frame. The change in the spacecraft's velocity in its own inertial frame (dv'), can be related to the change in velocity observed in the earth's inertial frame, dv by using eq. 3. The relationship between dv and dv' is

$$dv = \left(1 - \frac{V^2}{c^2}\right) dv' \quad (4)$$

where V is the instantaneous velocity of the spacecraft and $\frac{v'}{c} \ll 1$.

Conservation of Momentum

The change in the spacecraft's velocity (dv') in the spacecraft's inertial frame can be calculated using conservation of momentum. If a spacecraft of mass M annihilates a mass dm (e.g., an e^+e^- pair) it will emit each of the photons with a momentum $p' = (dm/2)c$.

Conservation of momentum in the spacecraft's iner-

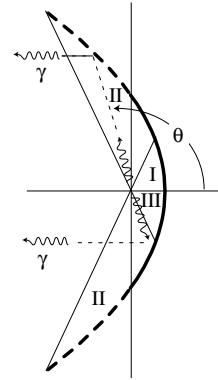


Fig. 2 Parabolic shield that ideally reflects gamma ray photons

tial frame can be written as

$$0 = Mdv' - p' \quad (5)$$

where one of the photons strikes the parabolic absorber at the end of the spacecraft. For this calculation, it is assumed that the parabolic dish (Fig. 2) extends from $-\frac{\pi}{2} \rightarrow \frac{\pi}{2}$ such that only one of the two photon impacts the dish. Furthermore, we initially consider the more realistic case where the photon is completely absorbed. To obtain the momentum component aligned with the direction of motion of the spacecraft, the quantity p' should be multiplied by $\langle \cos \theta \rangle$ which is:

$$\langle \cos \theta \rangle = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \cos \theta d\theta = \frac{2}{\pi} . \quad (6)$$

If the photon could be reflected, the $\langle \cos \theta \rangle$ term would be modified to include the recoil momentum of the photon. In the perfectly reflecting case where the parabolic dish extends from $-\frac{\pi}{2} \rightarrow \frac{\pi}{2}$, $\langle \cos \theta \rangle$ becomes $1 + \frac{2}{\pi}$. Returning to the case where the photon is absorbed and not reflected, Eq. 5 can be written as:

$$0 = Mdv' - \left(\frac{dm}{2}\right) c \times \frac{2}{\pi} . \quad (7)$$

Solving for dv' , we find that $dv' = \frac{dm}{M} \left(\frac{c}{2}\right) \left(\frac{2}{\pi}\right)$. Using the relationship $dm = -dM$ we can rewrite dv' as

$$dv' = -\frac{dM}{M} \left(\frac{c}{2}\right) \left(\frac{2}{\pi}\right) \quad (8)$$

Combining Eq. 8 with Eq. 4, we can finally write the change in the spacecraft's velocity (measured in the earth's inertial frame) as a function of the propellant mass expended in the spacecraft.

$$dv = -\left(1 - \frac{v^2}{c^2}\right) \times \frac{c}{2} \times \frac{2}{\pi} \left(\frac{dM}{M}\right) \quad (9)$$

This differential equation can be solved by separation of variables.

$$\int_0^v \frac{dv}{\left(1 - \frac{v^2}{c^2}\right)} = -\frac{c}{2} \frac{2}{\pi} \int_{M_i}^{M_f} \frac{dM}{M} \quad (10)$$

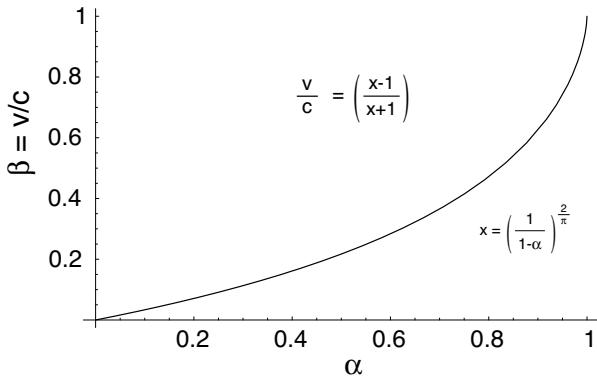


Fig. 3 Relativistic velocity of a spacecraft where α is the fraction of the spacecraft's mass used for e^+e^- annihilation. This assumes that the final state γ rays are completely absorbed on a parabolic dish extending from $-\pi/2 \rightarrow +\pi/2$.

Integrating both sides, the following relation is obtained:

$$\ln\left(\frac{c+v}{c-v}\right) = \frac{2}{\pi}\ln\left(\frac{M_i}{M_f}\right). \quad (11)$$

A running parameter α is defined to relate the initial and final masses of the rocket ($M_f = (1-\alpha)M_i$), where $0 \leq \alpha < 1$. The parameter α represents the fraction of the spacecraft's mass that is used for fuel (i.e., the mass fraction of e^+e^- pairs to produce the desired thrust). Substituting this into Eq. 11, the velocity as a function of fuel mass is

$$v = c \left(\frac{x-1}{x+1} \right) \quad (12)$$

where

$$x = \left(\frac{M_i}{M_f} \right)^{\frac{2}{\pi}} = \left(\frac{1}{1-\alpha} \right)^{\frac{2}{\pi}} \quad (13)$$

The velocity of the spacecraft as a function of α is shown in Fig 3.

Totally reflecting parabolic dish

To investigate the velocity profile for a totally reflecting parabolic dish, the $\langle \cos \theta \rangle$ term must be recalculated. Equation 6 is modified to include reflected γ rays in the following way:

$$\langle \cos \theta \rangle = \frac{1}{\Delta\theta} \int_0^\theta (1 + \cos \theta) d\theta \quad (14)$$

where the factor of "1" takes into account the momentum transfer along the axis of symmetry due to a perfectly elastic recoil of the γ ray.

To continue this calculation, the parabola is extended to include the "dashed" portion (Fig. 2) to maximize the number of γ rays reflected from the parabola. Furthermore, the parabola is split into three regions as shown in Fig. 2. In regions I and III, only one γ ray strikes the parabolic dish. If one of the γ rays is reflected in region II, then both photons will

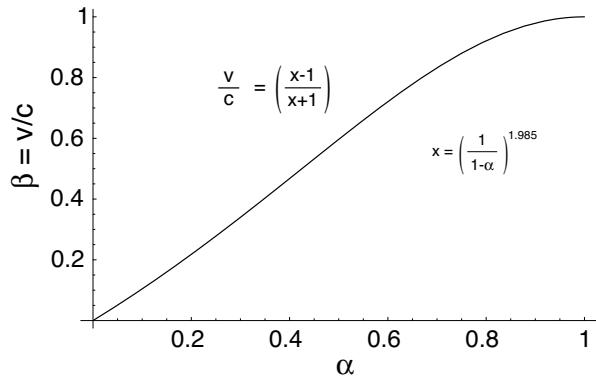


Fig. 4 The velocity as a function of α , the mass fraction used for fuel. This is for a parabolic reflector that extends from $-5\pi/6 \rightarrow +5\pi/6$.

strike the parabolic dish, thus enhancing the performance of the engine. Summing up the contributions from all three regions for a parabola extended from $-5\pi/6 \rightarrow 5\pi/6$, the factor $\langle \cos \theta \rangle$ becomes 1.985, where 2.000 would represent a parabola extending out to infinity. Substituting this factor into Eq. 7 in place of the $2/\pi$, and carrying out the same calculation as before, the velocity becomes

$$v = c \left(\frac{x-1}{x+1} \right)^{1.985} \quad (15)$$

where

$$x = \left(\frac{M_i}{M_f} \right)^{1.985} = \left(\frac{1}{1-\alpha} \right)^{1.985}. \quad (16)$$

The velocity as a function of α is shown for the extended parabolic reflector in Fig. 4. As expected, the velocity profile for the completely reflecting parabola (Fig. 4) produces faster velocities with less antimatter when compared to the non-reflecting parabola (Fig. 3).

Spacecraft performance

A spacecraft equipped with a matter-antimatter engine will have a specific impulse on the order of 10^7 s. Because of the large power output from the matter-antimatter fusion, it is possible to create many meganewtons of thrust.⁵⁻⁷ Theoretically the thrust is limited by the flow rate of positrons and electrons into the focal point. If the e^+ and e^- injection occurs at velocities less than 250 m/s, positronium atoms will form and annihilate with a high efficiency. This results in a mass injection rate \dot{m} of 4.03 gm/s.

From Newton's second law, the thrust can be related to the specific impulse. The relationship between force and change in momentum can be simply written as

$$F = \dot{p} = Ma \quad (17)$$

where M is the instantaneous mass of the spacecraft and a its acceleration. For the purpose of this discussion, it is assumed that the acceleration, a , is constant.

Since the rate-of-change in momentum is derived from photons, it is also possible to relate Eq. 17 to the force of a photon, namely

$$F = \dot{p} = \frac{\dot{E}}{c} \quad (18)$$

where \dot{E} is the power transferred to the spacecraft. To calculate the fraction of energy that ultimately propels the rocket, the ratio $1.985/2$ must be included (i.e., the $\langle \cos \theta \rangle$ term for the extended parabola). The 1.985 factor is due to the large fraction of photons being reflected off the extended parabola ($-5\pi/6 \rightarrow +5\pi/6$), while the factor of 2 represents an idealized situation where all the photon energy is captured by a parabola extending from $-\pi \rightarrow \pi$, a physical impossibility. Using Einstein's relationship between mass and energy, the power along the axis of symmetry can be written as

$$\dot{E}_{\parallel} = \left(\frac{1.985}{2} \right) \dot{m}c^2. \quad (19)$$

Combining equations 17-19 the force along the axis of symmetry becomes

$$F_{\parallel} = 0.9925 (\dot{m}c). \quad (20)$$

If 4.03 gm/s of electrons and positrons are annihilated at the focus of the parabola, this results in a power output of 3.59×10^{14} J/s. Substituting this value into Eq. 20, the total photon force is 1.20 MN. This value of the thrust can be used in Eq. 17 to obtain the thrust, and then used to determine the specific impulse by using the following equation

$$I_{sp} = \frac{F}{\dot{m}g} \quad (21)$$

where \dot{m} is the mass flow rate of electrons and positrons and the acceleration of the spacecraft is held constant $\sim g$. Combining Eqs. 20 and 21, the specific impulse becomes

$$I_{sp} = \frac{0.9925 c}{g} . \quad (22)$$

Substituting values for c and g , the specific impulse is calculated to be 3.04×10^7 s. This is the specific impulse for a completely reflecting parabolic dish that extends from $-5\pi/6 \rightarrow +5\pi/6$.

Mission Parameters

To best display the advanced performance of an APD, the minimum times of flight for rendezvous with Mars and Jupiter are calculated. To calculate the travel times for both missions, a Direct Trajectory Optimization Method (DTOM) trajectory code was used.⁸ In both cases the spacecraft starts with $C_3 = 0$ (i.e., a heliocentric orbit with the same position and

velocity vectors as the earth). In both cases, the spacecraft's orbit is transferred to heliocentric orbits of the respective planets (i.e., $C_4 = 0$ and $C_5 = 0$).

Further energy is required to insert the spacecraft into a planetary parking orbit. Although the ΔV needed to insert the spacecraft into this orbit is not shown in the following results, the energy and time required is insignificant compared to the rest of the mission.

Planet	M_{rocket}	$M_{propellant}$	Travel time
Mars	400 Mt	1.336 Mt	3.84 days
Jupiter	400 Mt	3.765 Mt	10.8 days

Table 3 Calculated travel times for a Martian and Jovian rendezvous assuming a propellant mass flow rate of 4.03 gm/s.

In Table 3 it is shown that the spacecraft can reach Mars and Jupiter in as little as 3.84 and 10.8 days respectively. Although this is very time efficient, slower velocities and longer travel times would still be acceptable.

While this is well-suited for travel within our solar system, the APD would also be capable of interstellar missions.^{1,3} Unlike the rendezvous times within the solar system, the spacecraft must coast for most of the trip to efficiently use its fuel. The time required to travel to Alpha Centauri at relativistic velocities is shown in Table 4. If 90% of the mass is used for fuel, then one-way travel times of 5.12 years can be achieved. Likewise, astronaut aging is significantly reduced to 1.65 years for the same trip.

M_{rocket}	$M_{propellant}$	Velocity	t_t (years)	t_o (years)
400 Mt	53.9 Mt	0.10 c	45.7	45.5
400 Mt	170 Mt	0.50 c	9.59	8.41
400 Mt	360 Mt	0.98 c	5.12	1.65

Table 4 Calculated travel times for a journey to Alpha Centauri using the APD engine. The time intervals for an earth observer (t_t) and an astronaut (t_o) are shown for three different fuel fractions. The difference between t_t and t_o are due to relativistic effects.

Conclusion

With the use of an APD engine, it is foreseeable that in the mid-to-distant future, relativistic manned interplanetary and interstellar travel is technically feasible. One of the major hurdles to overcome will be the technology required to produce and store large masses of charged antimatter.^{9,10} While this has been accomplished for small quantities of antimatter, any near-term solutions will require breakthrough technologies that are capable of storing many kilograms of positrons or antiprotons.

References

- (1) Gaidos, G., Lewis, R.A., Meyer, K., Schmidt, T., Smith, G.A. (1998). *AIMStar: Antimatter Initiated Microfusion for Pre-cursor Interstellar Missions*. Conference on Applications of Thermophysics on Microgravity and Breakthrough Propulsion Physics. STAIF-99 Albuquerque, NM. January 31- February 4, 1999.
- (2) Gerrish, H.P., Martin, J.J., Meyer, K.J., Schmidt, G.R., Smith, G.A. (1999). *Antimatter Production for Near-term Propulsion Applications*. Laboratory for Elementary particle Science. Pennsylvania State University. University Park, PA.
- (3) Chakabarti, S., Dundore, B., Gaidos, G., Lewis, R.A., Smith, G.A., (1998). *Antiproton-Catalyzed Microfission/fusion Propulsion Systems for Exploration of the Outer Solar System and Beyond*. The 34th Joint Propulsion and Power conference, AIAA Paper 98-3589, Cleveland, OH. July 13-15, 1998.
- (4) Kammash, Terry (1995). *Fusion Energy in Space Propulsion*. Washington, D.C.: American Institute of Aeronautics and Astronautics.
- (5) Brown, Charles (1992). *Spacecraft Mission Design*. Washington D.C.: American Institute of Aeronautics and Astronautics.
- (6) Hill, Philip G. (1992). *Mechanics and Thermodynamics of Propulsion*. (2nd edition) New York: Addison-Wesley Publishing Company.
- (7) Wiesel, William (1989). *Spaceflight Dynamics*. New York: McGraw-Hill, Inc.
- (8) Kluever, D.A., (1997). *Optimal Low-Thrust Interplanetary Trajectories by Direct Method Techniques*, Journal of Astronautical Sciences, Vol. 45, No. 3, pg. 247-267.
- (9) Cassenti, Brice (1000). *Mass Production of Antimatter for High Energy Propulsion*. AIAA, Journal of Propulsion and Power, Volume 16, Number 1.
- (10) Hotzscheiter, M.H., Lewis, R.A., Mitchell, E., Rochet, J., Smith, G.A. (1999). *Production and Trapping of Antimatter for Space Propulsion Applications*. Laboratory for Elementary Particle Science. Pennsylvania State University. University Park, PA.